

Restructuring Failure and Optimal Capital Structure

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August 2013

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Abstract

I build a dynamic capital structure model that allows the firm to renegotiate debt with its creditors. Renegotiations between creditors and equity holders are not always successful as debt forgiveness by some creditors increases the value of other creditors' debt claims. Rationally anticipating that the firm's assets are insufficient to cover the creditors' claims under these externalities debtholders refuse to participate in a restructuring and the firm is inefficiently liquidated. The probability of successful renegotiations increases in the value of the firm's assets at the time of restructuring, the concentration of the debt structure, and in the costs of liquidating the firm's assets. Anticipating the outcome of the debt restructuring I solve for the firm's optimal capital structure in a dynamic tradeoff model. Contrasting the classical tradeoff theory optimal leverage is non-monotonic in bankruptcy costs. When bankruptcy costs are low and debt is held by multiple creditors renegotiations will fail and optimal leverage is decreasing in bankruptcy costs in line with the trade-off theory. High bankruptcy costs increase the probability that renegotiations succeed making debt more attractive resulting in higher optimal leverage. Firms with low bankruptcy costs will optimally have a concentrated debt structure while firms with high bankruptcy costs maximize ex-ante firm value with dispersed debt.

Preliminary and incomplete

1 Introduction

Ni iudicatum facit aut quis endo eo in iure vindicit, secum ducito, vincito aut nervo aut compedibus XV pondo.

Unless the debtor pays the amount of the judgment or somebody guarantees his debt the creditor shall take him home and fasten him in stocks or fetters. He shall fasten him with fifteen pounds of weight.

The twelve tables - Roman code of law (451-450 B.C.)

To avoid dead weight losses of bankruptcy it is more efficient to renegotiate the debt of a financially distressed creditor, yet not all firms can successfully restructure their debt and have to enter an often costly bankruptcy process. The failure of debt restructurings is often assumed to be due to a coordination failure of myopic creditors, who are not willing to participate in the restructuring process. However, debt restructurings often succeed even in the presence of dispersed debt. This paper explores the conditions under which debt restructurings with multiple creditors can be successful and their implications on the firms optimal leverage and debt structure.

I model bankruptcy as an endogenous outcome of a bargaining game where the creditors fail to reach an agreement. Bargaining between multiple rational creditors in a setting with complete information can break down in which case the firm has no other option than to go to bankruptcy court. Bargaining frictions occur because creditors cannot all simultaneously agree to forgive debt. Specifically I assume that the proposer approaches creditors sequentially to sign them up for the restructuring plan. Creditors then optimally decide to join the restructuring plan or to evoke the bankruptcy mechanism. Creditors that forgive debt put themselves at a disadvantage relative to other creditors and thus increase the value of subsequent creditors' claims. Creditors that negotiate later can thus demand more for surrendering their existing debt claim. Anticipating the demands of future creditors the first creditors will only agree to a restructuring whenever the firms assets are large enough to cover the demands of all the firms creditors.

The efficiency of the bankruptcy process as the next best alternative to debt renegotiations will drive chances of a successful restructuring. When bankruptcy costs are low creditors see the bankruptcy court as a good alternative especially when other creditors have already reduced their debt. Anticipating that not all creditors are willing to participate in the restructuring the

other creditors decline as well and the firm ends up in bankruptcy. High bankruptcy costs reduce the creditors outside options and the prospect of small payouts in bankruptcy facilitate renegotiations.

Another decisive factor of successful restructurings is the number of creditors. While bargaining between one creditor and the equityholders is always successful a larger number of creditors increases the chances for bargaining failure. As each creditor agrees to a reduction in debt they make subsequent creditors' existing debt claims more valuable. While a larger number of creditors reduces the marginal impact that each creditors debt forgiveness has on subsequent creditors claims it still allows each subsequent creditor to extract larger premium for surrendering their existing debt claim. With a continuum of infinitesimal small creditors renegotiations can fail or succeed depending on bankruptcy costs and the value of the firms assets at the time of restructuring.

Finally the value of the firms assets at the time of restructuring contributes to the success of renegotiations. With more assets to share amongst creditors renegotiations will be successful for a variety of bankruptcy costs and debt structures. When the firms assets are worth little renegotiations are more likely to fail and bankruptcy costs and the debt structure are more decisive in determining the success of renegotiations.

The firm would thus like to commit ex ante to start renegotiations at the first sign of financial distress, however the option value that the equity holders have due to limited liability create an incentive to defer renegotiations. To solve for the optimal restructuring threshold at which equityholders start renegotiations with the debtholders I embed the restructuring game in a standard continuous time capital structure model. Anticipating the outcome of the following bargaining game the equityholders initiate renegotiations when their marginal payoff under the restructuring plan equals the marginal payoff of keeping the firm alive. The equityholders optimal choice of the restructuring threshold will again influence the outcome of the restructuring process.

The possibility to renegotiate debt thus determines not only the optimal restructuring threshold but also the firm's optimal ex-ante capital structure choice. Successful restructuring will affect the recovery of bondholders and thus the price at which the equityholders can sell the bonds. Successful renegotiations will also reduce dead weight bankruptcy costs which will be ex-ante born by the equityholders. I find that optimal leverage is non monotonic in bankruptcy costs. Consistent with the trade-off theory optimal leverage is initially decreasing in bankruptcy costs.

However, as bankruptcy costs increase renegotiations are more likely to be successful which increases the attractiveness of debt. Optimal leverage thus increases in bankruptcy costs when higher bankruptcy costs allow firms to successfully renegotiate debt. I also show that firms with low bankruptcy costs optimally seek a concentrated debt structure to facilitate renegotiations while firms with high bankruptcy costs optimally choose dispersed debt.

This paper contributes to a large literature on debt renegotiations. Haugen and Senbet (1978) point out that bankruptcy costs should not influence capital structure decisions when debt can be renegotiated. Bolton and Scharfstein (1996) examine in their seminal paper the optimal debt structure and renegotiations when an entrepreneur can default strategically. This paper abstracts from managerial agency problems and focuses on the implications of renegotiation failure on optimal capital structure. Gertner and Scharfstein (1991) examine renegotiations with one bank and with myopic dispersed debt-holders. In this paper we examine a multilateral bargaining game where each player behaves strategic and considers the impact of their action on the overall outcome. More recently Gennaioli and Rossi (2013) examine the optimal allocation of liquidation control rights and collateralization between a large and dispersed small bondholders. I focus more on the conditions under which successful renegotiations are possible and the implication for optimal capital structure.

My research also ties in a large body of literature on optimal dynamic capital structure including amongst others Fischer, Heinkel, and Zechner (1989), Goldstein, Ju, and Leland (2001), and Leland and Toft (1996). While all these papers assume that the firm gets liquidated when equity-holders walk away, in my model equity-holders optimally choose the point at which to initiate renegotiations with bondholders that may lead to positive payments to equityholders. Glover (2012) finds that firms with high bankruptcy costs take on less debt as equityholders have to pay ex-ante for the expected cost of default. My model adds an opposing effect because high bankruptcy costs increase the likelihood of successful renegotiations. This paper is closely related to Christensen, Flor, Lando, and Miltersen (2012) who model renegotiations with one creditor in a continuous time capital structure model. Because of the multilateral bargaining, renegotiations in this paper are not always successful which has important implications for capital structure and the optimal number of creditors.

The rest of the paper is organized as follows: Section 2 describes the bargaining game, Section 3 describes the optimal capital structure choice, the implications for the optimal debt

structure is analyzed in Section 4, and Section 5 concludes.

2 Bargaining and debt renegotiations

Let us start with assuming that equityholders have decided to enter renegotiations with the creditors for a voluntary restructuring of the firm's debt and analyze the outcome of the resulting bargaining game. Whether renegotiations are successful or not and the payoffs for the equityholders from a restructuring will then determine the restructuring threshold at which equityholders optimally start renegotiations, which we will examine in detail in Section 3.

Denote the market value of the firms assets at this restructuring threshold by \underline{v} and the face value of the outstanding debt D_f . Before exploring the bargaining process in detail we have to specify what happens when debtors and equityholders cannot reach an agreement and a voluntary restructuring fails.

2.1 The bankruptcy mechanism

When claimants of the firm are unable to reach an agreement every player has the option to go to bankruptcy court where a pre-specified bankruptcy mechanism gets implemented. We interpret this mechanism as liquidation but it can also be seen as the outcome of a chapter 11 reorganization which is handed down by the bankruptcy judge. In liquidation a fraction α of the assets gets destroyed and the value of the remaining assets get allocated proportionally to the creditors. We can interpret α either as direct bankruptcy costs that arise from going to court, e.g. for paying lawyers and accountants, or as indirect costs that arise when assets are liquidated.

Under the bankruptcy mechanism equityholders get zero. Assume that there are n debtholders and denote the face value of creditor i at the time that the bankruptcy mechanism gets evoked with x_i . Then the payoff for creditor i in liquidation is

$$L_i = \frac{x_i}{\sum_{j=1}^n x_j} (1 - \alpha) \underline{v} \quad (1)$$

The payoffs for equityholders and all debtholders at the restructuring threshold are then

$$\pi_L^D = \sum_{i=1}^n L_i = (1 - \alpha)\underline{v} \quad (2)$$

$$\pi_L^E = 0 \quad (3)$$

2.2 Bargaining protocol

In renegotiations equityholders bargain with n debtholders over a restructuring plan for the firm. I assume that the proposer cannot sign up all players to the restructuring plan simultaneously. To model sequential agreement to a restructuring plan assume that nature selects a sequence in which players arrive at the bargaining site. The first player to arrive is the proposer who makes a take-it-or-leave-it offer to claim-holders to exchange their existing securities of the firm for new securities. Any claim-holder can refuse the proposer's offer and go to bankruptcy court, where the bankruptcy mechanism gets implemented. If, after the last player has arrived, the value of the firm's assets is at least as high as the aggregate value of the outstanding securities the restructuring is successful, otherwise the firm goes to bankruptcy court and the bankruptcy mechanism gets implemented. Our mechanism is in spirit very similar to a voluntary debt restructuring. Under the Trust Indenture Act of 1939 in the US any change in the interest rate, the principal amount, or the maturity of public debt in an out of court restructuring requires an unanimous vote, so in practice debt restructurings are often accomplished by exchange offers.¹

We assume that with probability γ the equityholders arrive first at the bargaining site and offer subsequently arriving debtholders to redeem their debt claim for a new bond with reduced principal and with probability $1 - \gamma$ one of the debtholders arrives first and offers subsequent claimants new securities on the firm. To model a friction in our bargaining framework we assume that bondholders that agree to reduce their claim in a restructuring reduce the amount that they can claim in a subsequent bankruptcy compared to bondholders who hold out. Holdout problems are widely blamed for a the failure of out of court debt restructurings. Jensen (1991) argues that institutional changes have reduce the chance of private debt restructurings to succeed. In January 1990 bankruptcy Judge Burton Lifland ruled in the case of LTV corp. that bondholders who participated in a voluntary restructuring before the bankruptcy could only

¹see e.g. Hotchkiss, John, Mooradian, and Thorburn (2008).

claim a reduced value in bankruptcy compared to bondholders that held out.² Empirically Gilson, John, and Lang (1990) find that 53% of the firms in their sample fail to privately restructure their debt and subsequently file for bankruptcy.

We use this friction to model the holdout problem in a simple way. Creditors that make a concession to the company and reduce the face value of their debt claim decrease the fraction of the assets liquidation value that they are entitled to receive under the bankruptcy mechanism. At the same time the fraction that the other debtholders can claim increases. It is easy to see from equation (1) that the liquidation payoff of creditor i increases when creditor j has forgiven more debt (x_j decreases). Debt forgiveness by one creditor therefore generates a positive externality for the other creditors by increasing the payoff that they can get when bargaining fails. This externality increases in the threatpoints of the other players, i.e. the value that players can extract by rejecting the proposer's offer and thus invoking the bankruptcy mechanism. Each creditor who arrives at the bargaining site will get a higher payoff from evoking the bankruptcy mechanism than the creditor before him. The proposer has to offer each creditor more than the previous one and there could be a situation in which the value of the firm's assets are insufficient to meet the increasing demands of creditors. All creditors anticipate the demands of subsequent creditors and if assets are insufficient to meet aggregate creditor demand the first creditor will reject the proposer's offer and thus invoke the bankruptcy mechanism. In this case bargaining breaks down and the firm's debt cannot be successfully renegotiated.

Our bargaining model differs in two important aspects from traditional bargaining problems that can be solved using a characteristic function and Shapley values: first, the payoff that one group of players can obtain is not independent of the other players' actions because of the positive externality that a player's concession creates for the other players' minimum payoff. Second, our value function can be non-monotonic in the size of the bailout coalition, i.e. a creditor can be better off by not joining a bailout coalition (i.e. evoking the bankruptcy mechanism) than by joining and thus agreeing to a restructuring. This possible non-monotonicity in the value function and the presence of externalities prevent us from applying standard solution concepts of multi-player bargaining theory like the Shapley value in our case. Using the sequential arrival order imposes enough structure to ensure that a well defined equilibrium always exists.

²Even though that ruling was ultimately overturned in April 1992 for this specific case, there is substantial concern amongst bankruptcy professionals regarding the legal uncertainty of restructured claims in subsequent bankruptcies (Betker (1995)).

2.3 Renegotiations with one creditor

With one creditor renegotiations of the firm's debt are fairly straightforward because there cannot be any externalities on other creditors. As the equityholders enter renegotiations the firm's assets are worth \underline{v} . If the equityholders propose first, which happens with probability γ they will offer the debtholder a payment just above what the debtholder would get under the bankruptcy mechanism, i.e. L_1 . The debtholder will accept as he cannot improve his payoff by rejecting the offer and thus invoking the bankruptcy mechanism. If the debtholder can propose first, he will offer the equityholder a payment just above zero, which the equityholders will accept as they get zero in bankruptcy court. Renegotiations will always be successful and the bankruptcy mechanism will never be evoked in equilibrium. The following proposition summarizes this finding.

Proposition 1 *Renegotiations with one creditor are always successful and the bankruptcy mechanism will never be evoked in equilibrium. Given that the assets of the firm are worth \underline{v} the expected payoffs for the debtholder and the equityholders given one creditor are $\pi_1^D = (1-\gamma)\alpha\underline{v}$ and $\pi_1^E = \gamma\alpha\underline{v}$, respectively.*

2.4 Renegotiations with many creditors

With many creditors bargaining might fail and the only equilibrium is the liquidation of the firm under the bankruptcy mechanism. Consider the following example:

Example 1 *Assume that upon entering renegotiations the firm has assets worth $\underline{v} = 100$. Assume furthermore that the firm has two creditors with claims of $d_1 = d_2 = \$60$ each and that liquidation costs are $\alpha = 5\%$. Assume that the equityholders propose first.*

If the first creditor that arrives at the bargaining site refuses the equityholders offer the company ends up in bankruptcy court at which point the first creditor will get from Equation (1) a liquidation payoff of $L_1 = \frac{d_1}{d_1+d_2}(1-\alpha)\underline{v} = \frac{60}{60+60}(1-0.05)100 = 47.5$. The equityholders thus have to offer the first creditor at least $x_1 = 47.5$. The concession of the first creditor, however, creates a positive externality for the second creditor as he arrives at the bargaining site. If he refuses the

equityholders' offer he will get in liquidation $L_2 = \frac{d_2}{x_1+d_2}(1-\alpha)v = \frac{60}{47.5+60}(1-0.05)100 = 53.02$. The second creditor will therefore not accept any offer that is below \$53.02. Such an offer, however, is infeasible for the equityholders as the sum of the minimum acceptable offers for both creditors exceeds the firm's resources, $53.02 + 47.5 = 100.52 > 100$. Anticipating that the equityholders cannot make the second creditor an acceptable offer the first creditor refuses any offer from the equityholders and goes to bankruptcy court.

The example illustrates two necessary conditions for a breakdown of bargaining: first, the asset value can not be too high. In this particular example the two debtors can agree on a bargaining solution as long as the asset value upon entering renegotiations exceeds \$102.25. It is easy to verify that in this case the optimal strategy of the equityholders is to offer \$48.57 and \$53.68 to the first and second creditor, respectively, and keep zero for themselves. Second, bargaining will only break down when liquidation costs are low. Low liquidation costs increase players' payoff under the bankruptcy mechanism and thus decrease their willingness to accept the proposer's offer. In the extreme case of 100% liquidation costs creditors' outside option is zero, they will therefore accept any offer that leaves them with a weakly positive payoff and renegotiations always succeed. In example 1 creditors can find a bargaining solution as long as liquidation costs $\alpha > 5.6\%$. The intuition for the symmetric bargaining case with three players is summarized in the following proposition:

Proposition 2 *Assume that upon entering renegotiations the firm has assets worth v and two outstanding debt claims with face value $D/2$ each. Renegotiations will only fail when liquidation costs $\alpha < \frac{1}{3}$ and $v < v_3^* = \frac{D(1-3\alpha)}{1-\alpha^2}$. If renegotiations are successful the expected payoff for equityholders is*

$$\pi_3^E = \frac{1}{2}\gamma v \left(\alpha + \frac{2(\alpha-1)D}{D-\alpha v+v} + 1 \right) \quad (4)$$

and for each debtholder is

$$\pi_3^D = v - \frac{(\alpha-1)\gamma Dv}{D-\alpha v+v} - \frac{1}{2}(\alpha+1)\gamma v \quad (5)$$

From proposition 2 we can see that a seemingly more efficient bankruptcy mechanism with lower liquidation costs α can lead to a more inefficient outcome where a voluntary restructuring

of the firms debt is impossible and dead weight losses are realized under the bankruptcy mechanism. The critical asset value v_3^* under which renegotiations fail increases as liquidation costs fall, making it even harder for firms to restructure.

Our findings have also important empirical implications for estimating bankruptcy costs. Firms with higher bankruptcy costs will be able to restructure out of court and thus now show up in a sample of bankruptcy filings. This possible selection bias might lead to an underestimation of actual bankruptcy costs.

Proposition 2 also has important implication for the point at which equityholders optimally choose to restructure the firm. In the classic capital structure literature equityholders support the firm's debt as long as the call option value of keeping the firm alive is greater than the cost of the coupon payments. With renegotiations equityholders might collect a positive payoff in restructuring which will change the point at which they optimally decide to enter renegotiations. We will endogenize this lower restructuring threshold in Section 3 of this paper.

It is fairly easy to generalize our findings for the case of n players. We have to solve for the equilibrium numerically using the following procedure:

Proposition 3 *Let $\langle d_i \rangle$ be the sequence of face values of the n creditors' debt claims. Then renegotiations are successful as long as*

$$\sum_{i=1}^n x_i \leq \underline{v}, \quad (6)$$

where

$$x_i = \frac{d_i}{\sum_{k=1}^{i-1} x_k + \sum_{l=i}^n d_l} \quad (7)$$

The aggregate payoffs for equityholders and bondholders are

$$\pi_n^E = \gamma(\underline{v} - \sum_{i=1}^n x_i) \quad (8)$$

$$\pi_n^D = \underline{v} - \gamma(\underline{v} - \sum_{i=1}^n x_i), \quad (9)$$

respectively.

The case of diapered debt can be approximated by assuming a continuum of infinitesimal small debtholders. Even in this case successful renegotiations are possible when we assume that debtholders consider the impact of their decision on the overall outcome of the restructuring.

Proposition 4 *Assume that there exists a continuum of infinitesimal creditors with an aggregate claim of D . Renegotiations of the firms debt is successful if $c(1) > 1 - v/D$ where $c(\cdot)$ is the solution to the following differential equation*

$$c'(n) = 1 - \frac{1}{1 - c(n)}(1 - \alpha)\underline{v} \quad (10)$$

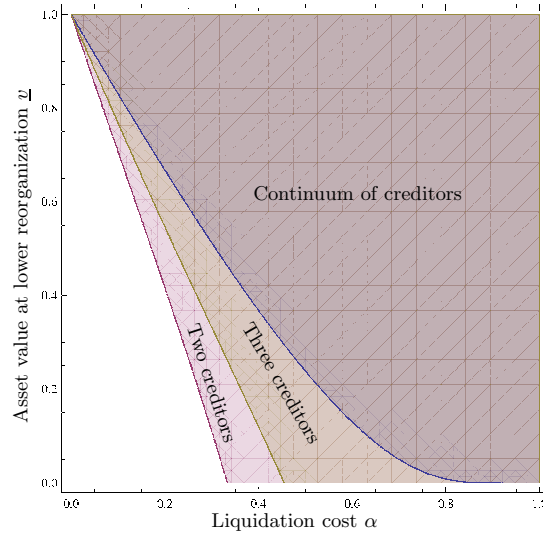
with the initial condition $c(0) = 0$.

Figure 1 illustrates the outcome of the renegotiation game graphically. When bankruptcy costs are low and the value of the firms assets is small compared to the outstanding debt then renegotiations will fail as soon as there is more than one creditor. Renegotiations are always successful when bankruptcy costs are high and the firm's assets are valuable. For intermediate regions of bankruptcy costs the debt structure and the the asset value are important. A more concentrated debt structure allows renegotiations to succeed. The success of renegotiations is also driven by the value of the assets that the firm still has. The decision to initiate renegotiations is clearly endogenous and the result of an optimal decision by the firm's equityholders. They must decide whether it is more advantageous for them to keep the firm going and risk inefficient liquidation in case that the assets deteriorate further in value or if they renegotiate with creditors given the expected outcome of the bargaining process. We model the equityholders decision and its implications on renegotiation outcomes and optimal capital structure in the next section.

3 Optimal capital structure

The solution to the renegotiations game defines payoffs conditional on the equityholders entering renegotiations but it does not define the point at which equityholders optimally start renegotiations with debtholders. As we saw from Section 2 the success of the bargaining process as well as its payoffs will depend on the value of the assets \underline{v} that can be shared. We can therefore not analyze renegotiations by themselves as the entry point is endogenous and optimally

Figure 1. Region of successful renegotiations in the case of two, three and a continuum of infinitesimal small creditors. The graph shows the region of successful renegotiations for different values of the liquidation costs α on the x-axis and the asset value \underline{v} on the y-axis. Debt is assumed to be 1. The largest area represents the case of two equally sized creditors, the middle sized area is for the case of three equally sized creditors and the smallest area is for the case of a continuum of infinitesimal small creditors.



chosen by equityholders to maximize their payoff. Depending on the number of creditors, firms bankruptcy costs, and their bargaining power, the equityholders will compare the expected payoff from the bargaining game with the value of their claim when the firm is kept alive. The choice of the optimal threshold at which to start renegotiations gets further complicated as the value of the firm's assets is partly driven by the possibility to renegotiate claims again in the future. We address this problem by embedding our bargaining model into a classical EBIT based continuous time model in the spirit of Goldstein, Ju, and Leland (2001) and solve for the optimal reorganization threshold.

The firm's EBIT, denoted by ξ , is exogenously created by the unique technology of the firm and is assumed to follow a geometric Brownian motion under the pricing measure

$$d\xi_t = \xi_t \mu dt + \xi_t \sigma dW_t \quad (11)$$

with initial value ξ_0 , constant drift μ and volatility σ .

The firm is controlled by the equityholders who issue perpetual, callable debt against the firm's EBIT. The debt pays an instantaneous coupon of c and can be called at any time at a proportional premium λ of the face value. Issuing debt incurs a proportional transaction cost of k . In line with the previous literature we assume that interest expenses are tax deductible resulting in a tax advantage of debt. Denote by r the constant risk free interest rate and the tax rates for interest income and dividend payments by τ_i and τ_e , respectively. The effective interest rate at which an investor can borrow to replicate contingent claims on the firm's EBIT is then $r(1 - \tau_i)$.³ We also assume that $\tau_e > \tau_i$ to generate a tax advantage of debt as a reason to issue debt.

When the firm's EBIT is either too high or too low, equity holders have an incentive to restructure the firm. Denote by ξ_0 the initial EBIT level. As the firm's EBIT grows the equityholders will find that the tax benefit at the current debt level is too low and may find it worthwhile to pay the transaction cost and re-lever the firm to the optimal debt level. At the upper restructuring threshold $u\xi_0$ equityholders call the outstanding debt at full face value and issue new debt. The upper restructuring threshold in our model is in line with the standard literature. As the firm's EBIT deteriorates the firm's cash flow is insufficient to maintain the coupon payments and equityholders have to inject funds to meet the debt obligations. At the lower restructuring threshold, denoted by the threshold $l\xi_0$, equityholders will find it optimal to walk away from the firm or initiate renegotiations on the firm's debt. We will determine the optimal location of the restructuring thresholds in Section 3.1

The value of the firm's debt and equity can be derived as contingent claims on the firm's EBIT. We follow closely the notation of Christensen, Flor, Lando, and Miltersen (2012) and similar to them we will show that the time t market value of debt $D(\xi_t, \xi_s)$ and equity $E(\xi_t, \xi_s)$ can be written as functions of the current EBIT level ξ_t and the EBIT level ξ_s at which the claims were issued given that the EBIT process has neither hit the upper nor the lower restructuring threshold. The pricing function for debt and equity will be derived in detail in Appendix A. It is also noteworthy that debt and equity are homogeneous of degree one in EBIT, e.g. $D(\kappa\xi_t, \kappa\xi_s) = \kappa D(\xi_t, \xi_s)$, which allows us to simplify notation such that the value of debt and equity at the time of issuance can be written as the product of a constant and the EBIT level at

³In line with the previous literature we have to assume that $\mu < r(1 - \tau)$ to ensure that the value of equity is finite.

the time of issuance:

$$\begin{aligned} D(\xi_s, \xi_s) &= \xi_s D(1, 1) = D\xi_s \\ E(\xi_s, \xi_s) &= \xi_s E(1, 1) = E\xi_s \end{aligned}$$

Finally denote by $A\xi$ the total value of the firm to the equityholders just before new debt is being issued. In addition to the equity value $E\xi$, the owners will issue debt with proceeds of $(1 - k)D\xi$ after issuance costs.

$$A\xi = E\xi + (1 - k)D\xi \quad (12)$$

The total face value of debt is $D\xi_0$ as we assume that debt is issued at par.

3.1 Restructuring Thresholds

As noted before the equityholders want to increase the firms leverage to take advantage of the tax shield when the EBIT process hits the upper restructuring threshold $u\xi_0$. At the upper restructuring threshold repayment of the firm's debt is imminent. When calling the issue the firm will pay debtholders a premium of λ over the face value $D\xi_0$ of the outstanding debt. The debt value has to satisfy the value matching condition

$$D(u\xi_0, \xi_0) = (1 + \lambda)D\xi_0. \quad (13)$$

Similarly the value of equity at the upper threshold is the value of equity immediately before the issuance of new debt has to equal the value of equity after the issuance plus the proceeds from the new debt issuance after transactions costs minus the repayment of the old debt including the call premium.

$$E(u\xi_0, \xi_0) = E(u\xi_0, \xi_0) + (1 - k)D(u\xi_0, u\xi_0) - (1 + \lambda)D(\xi_0, \xi_0) \quad (14)$$

$$= (Au - (1 + \lambda)D)\xi_0 \quad (15)$$

The pricing functions of debt and equity have to satisfy these two value matching conditions. To find the optimal upper restructuring threshold we have to solve the associated smooth pasting

condition. At the restructuring threshold the first derivative of equity with respect to EBIT immediately before issuance must equal the first derivative at issuance, i.e.

$$E^1(u\xi_0, \xi_0) = \frac{\partial A\xi}{\partial \xi} = A \quad (16)$$

where $E^1(x, \xi_0) = \left. \frac{\partial E(\xi, \xi_0)}{\partial \xi} \right|_{\xi=x}$ is the first partial derivative of the equity pricing function with respect to ξ .

At the lower restructuring threshold the equityholders can enter renegotiations with the debtholders to restructure the firm's debt as outlined in Section 2. We assume that the decision to initiate renegotiations is irreversible and that upon entering renegotiations a certain fraction ϕ of the assets gets destroyed, which we interpret as coming from advisory or legal fees, accounting costs, or managerial attention devoted to the renegotiation process. Once the equityholders have started bargaining the outcome and the payoffs are determined according to the bargaining game.

To determine bargaining payoffs we have to specify the value of the assets \underline{v} upon entering renegotiations. Any potential new buyers of these assets will again lever up the assets to the optimal leverage and thus generate extra value from the tax shield. At the lower restructuring boundary EBIT is $l\xi_0$ and the assets can be sold for the value of an optimally levered firm at that EBIT level. After subtracting the costs for entering renegotiation ϕ the value of the firms assets are thus

$$\underline{v} = (1 - \phi)Al\xi_0. \quad (17)$$

If this asset value at the lower restructuring threshold is greater than the face value of debt then debt is risk free as there are enough funds available at the restructuring threshold to pay off the firm's debt in full. Any remaining value would then go to the equityholders. In this paper we want to focus on the more interesting case when debt is risky and the asset value at the lower restructuring threshold is insufficient to satisfy the debtholders in full.⁴

The equityholders will determine the optimal lower restructuring threshold according to the

⁴We verify that debt is indeed risky for all the comparative statics results in the paper.

usual smooth pasting condition such that

$$\left. \frac{\partial E(\xi)}{\partial \xi} \right|_{\xi=l\xi_0} = \left. \frac{\partial \pi^E(\xi)}{\partial \xi} \right|_{\xi=l\xi_0} \quad (18)$$

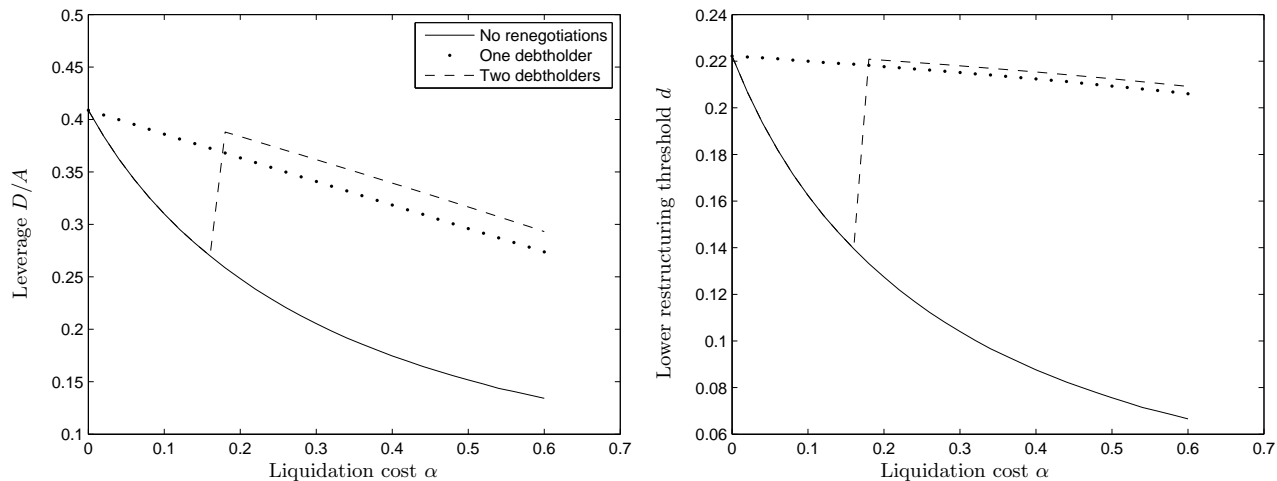
If we rule out renegotiations then equityholders always get zero at the lower restructuring threshold, $\pi^E = \pi_L^E = 0$, from Equation (3) and we get the standard result of the previous literature. In the general case we have to first determine whether or not renegotiations are successful as specified in Condition (6). If renegotiations fail, the firm gets liquidated and the payoff for equityholders is zero. If renegotiations are successful then equityholders can obtain a positive payoff and $\pi^E = \pi_n^E$ as specified in Equation (9).

Renegotiations have a profound impact on the lower restructuring threshold. When renegotiations are successful equityholders can obtain a positive payoff from restructuring the firms debt, which generates an incentive to renegotiate early. With two (or more) creditors the optimal thresholds at which equityholders initiate renegotiations is driven by bankruptcy costs. When bankruptcy costs are high, renegotiations are always successful and the renegotiation threshold only changes marginally as the payoffs from renegotiations change. When bankruptcy costs are low, however, two outcomes are possible: either equityholders start renegotiating early as long as the asset value is greater than V_2^* to ensure that renegotiations are successful. In this case equityholders might gain personally but this outcome can also be more efficient as the firm can save the liquidation costs that arise under the bankruptcy mechanism. The other alternative is that equityholders find it optimal purposely defer restructuring to exploit the call option of equity accepting that once the lower restructuring threshold is reached, renegotiations will fail and the bankruptcy mechanism will be evoked. In this case the optimal lower restructuring threshold will similar to the case without renegotiations. This solution can also be efficient when bankruptcy costs are low. Lowering the threshold EBIT level at which restructuring will occur results in less frequent renegotiations which saves the cost Φ of entering renegotiations and the issuance cost of the new debt.

To get some intuition for our model and show some comparative statics we solve our model for the following parameters which we refer to as the base case: $\mu = 0.02, \sigma = 0.25, \tau_i = 0.35, \tau_e = 0.45, r = 0.045, \lambda = 0.05, \alpha = 0.2, k = 0.03, \phi = 0.05, \gamma = 0.5$.

The left graph in Figure 2 plots the optimal initial leverage of the firm as a function of the

Figure 2. Optimal leverage (left) and lower restructuring threshold (right) as a function of liquidation costs without renegotiations, and with one and two creditors, respectively. All calculations are for the base case with the parameters $\mu = 0.02, \sigma = 0.25, \tau_i = 0.35, \tau_e = 0.45, r = 0.045, \lambda = 0.05, k = 0.03, \phi = 0.05, \gamma = 0.5$



liquidation cost α . Higher dead weight liquidation costs make debt financing less attractive for two reasons: first, more debt increases the probability of liquidation and this increases the probability that dead weight losses will be realized. Without renegotiation we see leverage decreasing sharply with bankruptcy costs. With one creditor renegotiations are always successful (as shown in Proposition 1) and the bankruptcy mechanism is never evoked and firms optimally choose a higher leverage for all levels of bankruptcy costs. With two (or more) creditors the optimal leverage is non-monotonic in bankruptcy costs. From Proposition 2 we know that renegotiations will not always be successful. Specifically for low bankruptcy costs renegotiations fail and the firm will always get liquidated. The optimal leverage then coincides with the case of no renegotiations. If bankruptcy costs are high enough renegotiations will always succeed and optimal leverage will jump to a much higher level. Renegotiation failure can thus have a profound impact on capital structure and lead to a non-monotonic relationship between liquidation cost and a firm's optimal capital structure.

The graph also shows that optimal leverage is declining in liquidation costs for all debtholder structures, even when renegotiations are always successful, no firm gets liquidated, and liqui-

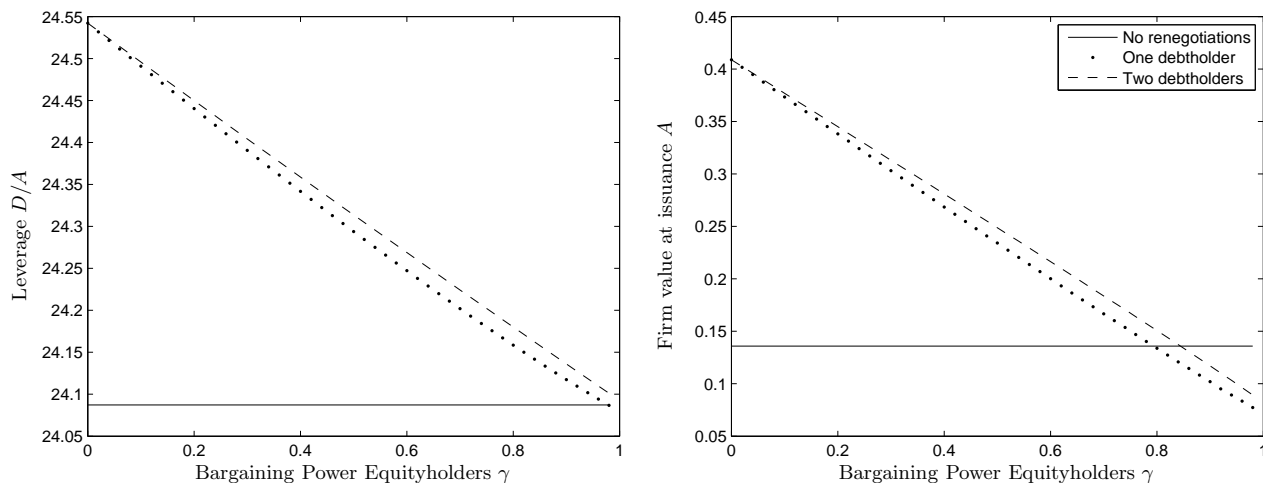
dation costs never occur. Increasing liquidation costs lowers the amount that debtholders obtain in bankruptcy, i.e. their threatpoint for bargaining, which decreases the amount that they can extract in renegotiations. Rationally anticipating that they will obtain less in renegotiations debtholders are willing to pay less for the bonds when the firm first issues its debt, making it less attractive for the initial owners to issue debt. This logic is also the reason why optimal leverage is higher with two debtholders than one conditional that renegotiations are successful. Two debtholders can in total extract more from a firm than one debtholder and thus are, *ceteris paribus*, willing to bid more for the firms newly issued debt. When initially the firms owners can sell debt at a higher price they will optimally issue a larger amount resulting in higher optimal leverage.

The optimal lower restructuring threshold is depicted in the right graph of Figure 2. After successful renegotiations the firm is re-levered to the optimal capital structure which increases firm value. When some of that value creation is shared with shareholders in a successful renegotiation of the firms debt, equityholders have an incentive to enter renegotiations early. We can see that equityholders enter renegotiations much earlier when they anticipate their success. Knowing that renegotiations will be unsuccessful equityholders take full advantage of their call option and wait longer to start restructuring the firm. We can also see that the lower restructuring threshold is declining in liquidation costs which is a direct result of the lower leverage that is associated with higher liquidation costs. With lower debt obligations equityholders find it worthwhile to support the firm for a longer time.

Bargaining and the debtor structure have an important implication on the location of the default barrier. A traditional structured model would set the default barrier much lower than it is with bargaining. By allowing the equityholders to participate in the restructuring gains they have an incentive to open renegotiations much earlier. The default barrier is thus much higher than traditional models would suggest. The structure of the firms debt, whether it is concentrated like in a European style bank based financial system or whether it is more dispersed as for the typical North American firm can also influence the optimal EBIT level at which firms enter renegotiations.

Figure 3 shows the optimal leverage and the overall firm value at the issuance of debt as a function of the relative bargaining power of equityholders γ . Giving more power to equityholders lowers the amount that debtholders can obtain in renegotiations. With a lower recovery

Figure 3. Optimal leverage (left) and overall firm value (right) as a function of bargaining power without renegotiations, and with one and two creditors, respectively. All calculations are for the base case with the parameters $\mu = 0.02, \sigma = 0.25, \tau_i = 0.35, \tau_e = 0.45, r = 0.045, \lambda = 0.05, k = 0.03, \phi = 0.05, \gamma = 0.5$

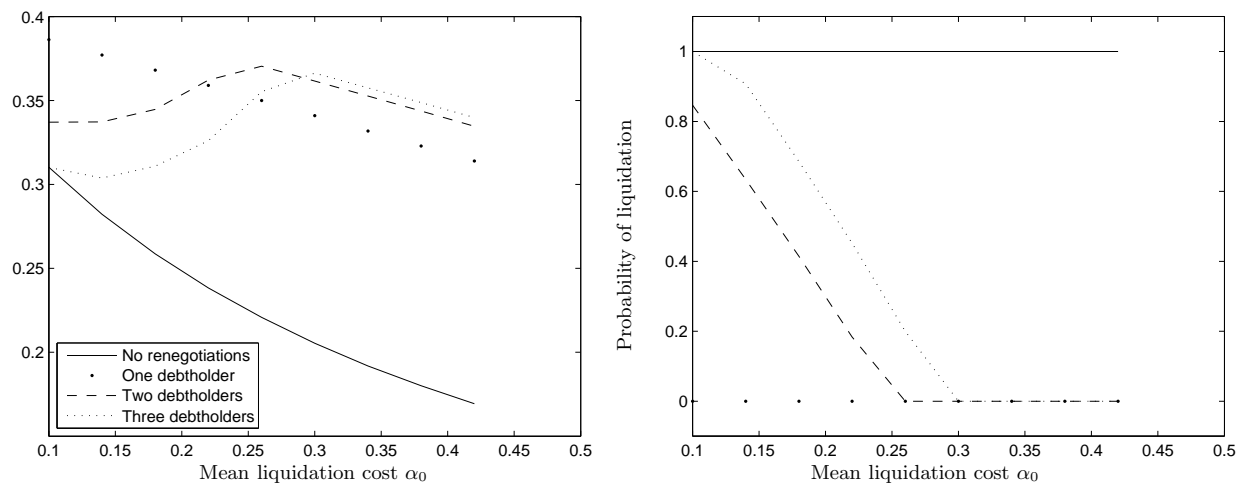


in renegotiations debtholders are demanding a higher coupon for a bond with the same face value. The firm can thus achieve the same interest tax shield with a lower face value of debt and therefore optimal leverage decreases as bargaining power of the equityholders increases. This change in capital structure also has a negative effect on overall firm value, which is decreasing in the equityholder's bargaining power. Our model has some interesting contributions to the optimal design of the restructuring and bankruptcy process. While it is important to allow parties to renegotiate their debt and to let equity holders participate in the gains of the restructuring gains it is not always optimal to allocate more bargaining power to the equityholders.

3.2 Stochastic bankruptcy costs

So far in our model equityholders can fully anticipate the outcome of the renegotiations with the firm's debtholders and optimally set the default boundary, the point at which they start renegotiations, to avoid liquidation of the firm if they find this to be optimal. In reality the outcome of renegotiations might not be known ex-ante and equityholders might have to initiate

Figure 4. Optimal leverage (left) and probability of liquidation (right) as a function of mean liquidation cost without renegotiations, and with one and two creditors, respectively. All calculations are for the base case with the parameters $\mu = 0.02, \sigma = 0.25, \tau_i = 0.35, \tau_e = 0.45, r = 0.045, \lambda = 0.05, k = 0.03, \phi = 0.05, \gamma = 0.5, d = 0.1$



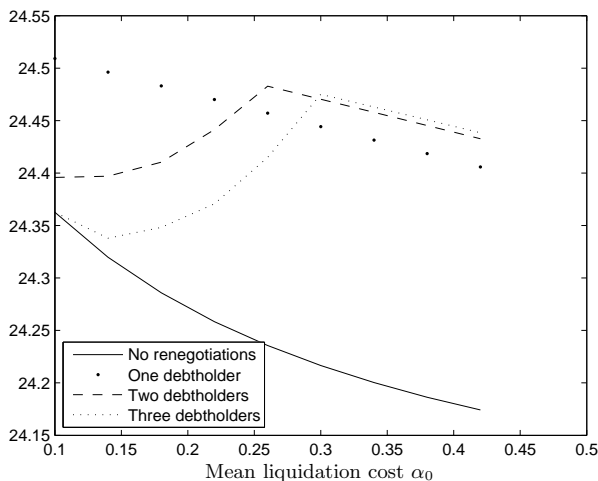
bargaining without certainty of success and their payoff. If the firm’s assets are industry specific it might for example be unclear at what price assets can be sold for or if the firm’s assets consist mainly of human capital it is unclear how many key employees will be lost in the restructuring process. We model this uncertainty about bargaining outcome by assuming that liquidation costs are random. Specifically we assume that liquidation costs α are drawn from a uniform distribution with mean α_0 once the firm enters the bargaining game:

$$\alpha \sim U[\alpha_0 - d, \alpha_0 + d]. \tag{19}$$

Figure 4 shows the optimal initial leverage as a function of average liquidation costs. Again the intuition from the model with fixed bankruptcy costs applies. Optimal leverage is a non-monotonic function in bankruptcy costs and optimal leverage declines in liquidation costs due to weakened bargaining position of the bondholders.

The right panel in Figure 4 shows the firms probability of getting liquidated under the optimal capital structure. Without renegotiations the firm gets always liquidated and with one

Figure 5. Firm value under the optimal capital structure as a function of mean liquidation cost without renegotiations, and with one and two creditors, respectively. All calculations are for the base case with the parameters $\mu = 0.02, \sigma = 0.25, \tau_i = 0.35, \tau_e = 0.45, r = 0.045, \lambda = 0.05, k = 0.03, \phi = 0.05, \gamma = 0.5, d = 0.1$



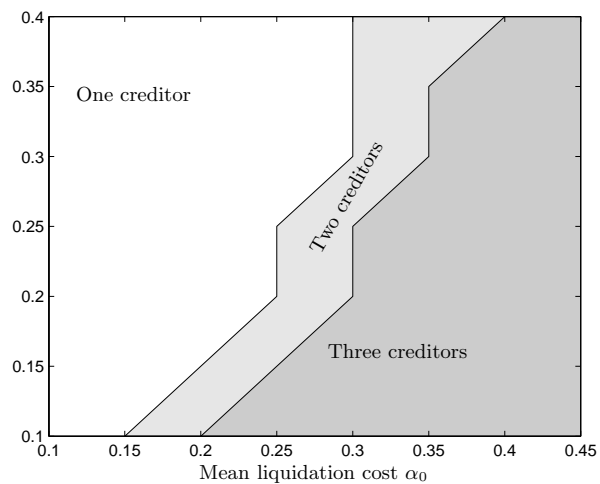
creditor renegotiations always succeed and the firm survives. For both, two and three debtors, the liquidation probability is decreasing in liquidation costs and in the number of debtors. Our model can thus explain the impact of debt structure on liquidation probabilities.

4 Debt structure

Since the number of creditors will affect the chances of success and the payoffs for claimants in renegotiations, debt structure will have an impact on firm value and depending on firm characteristics, our model can explain heterogeneity in firms' debtor structure. Figure 5 plots the firm value under the optimal capital structure for different values of average bankruptcy costs corresponding to the analysis in Figure 4. Firms with low bankruptcy costs optimally seek concentrated debt, with only one creditor for very low bankruptcy costs as renegotiations will always be successful under this debtor structure. Firms with higher bankruptcy costs have a higher chance of renegotiating debt and will optimally choose more dispersed debt.

Figure 6 shows the optimal debt structure as a function of mean liquidation costs and asset

Figure 6. Regions where one, two, and three creditors are optimal for a range of mean liquidation costs (x-axis) and asset volatilities (y-axis). All calculations are for the base case with the parameters $\mu = 0.02, \sigma = 0.25, \tau_i = 0.35, \tau_e = 0.45, r = 0.045, \lambda = 0.05, k = 0.03, \phi = 0.05, \gamma = 0.5, d = 0.1$



volatility. Concentrated debt is optimal for low bankruptcy cost high volatility firms while more dispersed debt is optimal for firms with low asset risk and higher liquidation costs.

5 Conclusion

To be completed

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A Pricing Functions for D and E

To be completed.

B Proofs.

To be completed.